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[1] The observation that the spin period of Venus is extremely close, although not equal, to the $p = -5$ spin-orbit resonance with the Earth makes it very improbable that such a situation is fortuitous. This leads one to explore hypotheses in which the Earth spin-orbit resonance plays some role in Venus's observed spin rate. This paper proposes one such hypothesis. Venus's core is assumed to be composed of a liquid outer core surrounding a solid inner core, the latter undergoing a 0.31 degree/year differential rotation with the mantle. Due to gravitational coupling, however, core-mantle differential rotation would be impossible, unless isostatic compensation exists with an effectiveness of 99.9998%. Within that assumption, it is proposed that Venus is trapped in the $p = -5$ spin-orbit resonance with the Earth, but that this resonance concerns the inner core rather than the mantle. Stable resonance requires that the inner core should depart significantly from spherical symmetry, while its material should still be able to sustain the stress differences produced by such asymmetric mass distributions. Compatibility between those two conditions is studied, leading to constraints on the size of the inner core. An appreciable probability of stable resonance is found to be achievable, provided that the average inner core radius is larger than a minimum, which is estimated as 1300 km within the heating at the ground hypothesis for atmospheric thermal tide. That condition would become considerably less stringent if solar heat absorption in the upper atmosphere rather than at the ground were assumed.

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1. Introduction

[2] The earliest observations of the slow, retrograde rotation of Venus suggested that it might be in a synodic spin-orbit resonance with the Earth [*Smith*, 1963; *Goldstein*, 1964; *Carpenter*, 1964, 1966, 1970; *Shapiro*, 1967]. Inside this resonance, the torque exerted by the Earth on the permanent bulge of Venus at each inferior conjunction would be always of the same sign, each of those contributions adding coherently and resulting in a net torque over the long term. Such a situation would correspond to the spin rate of Venus ω_R satisfying the following resonant condition [*Goldreich and Peale*, 1966]:

$$\omega_R - n_V = p[n_V - n_E] \tag{1}$$

where p is an integral multiple of 1/2 and n_E and n_V are the orbital mean motions of the Earth and Venus, respectively, which are given as $n_E = 2\pi/365.25636$ days, $n_V = 2\pi/224.70080$ days [*Bretagnon*, 1982]. The case of Venus may correspond to the resonance $p = -5$, for which equation (1) yields

$$\omega_R = \omega_5 = 5n_E - 4n_V = -2\pi/243.1650 \text{ days} \tag{2}$$

In the context of that resonance, Venus would complete exactly four retrograde rotations, as seen from the Earth (five, as seen from the sun) between two consecutive inferior conjunctions.

[3] *Gold and Soter* [1969] proposed a plausible explanation for the fact that such a comparatively weak terrestrial resonant torque could control the rotation of Venus. This would result from the fact that the two dominant torques (due to atmospheric and solid body solar tides) have opposite signs and have a different dependence upon Venus's solar day. Competition between the solar atmospheric and body tides resulted in a slow variation of the spin rate of Venus over the years, until ultimately both contributions became almost exactly opposite and canceled out each other. As Venus approached this equilibrium spin rate, it would have passed through several resonances and finally would have been trapped into the $p = -5$ resonance, which would be sufficiently close to the equilibrium spin rate. Following this approach, several authors studied the balance between atmospheric and body torques at Venus in more detail [e.g., *Ingersoll and Dobrovolskis*, 1978; *Gold and Soter*, 1979;

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Dobrovolskis and Ingersoll, 1980; *Correia and Laskar*, 2001, 2003; *Correia et al.*, 2003].

[4] As measurements of the spin rate of Venus became more precise, it turned out however that the observed spin rate ω_S of Venus was not exactly the resonance spin rate ω_R of equation (2). Early observations from terrestrial radars gave a (retrograde) spin period of Venus T_S = 243.01 days [*Shapiro et al.*, 1979]. *Slade et al.* [1990] obtained T_S = 243.022 ± 0.003, while *Davies et al.* [1992] obtained T_S = 243.0185 ± 0.0001 days from the analysis of Magellan radar images.

[5] *Bills et al.* [1987] pointed out that it would be necessary for the atmospheric angular momentum to fluctuate by ±40% (thus providing the required fluctuation of the body spin rate under conservation of momentum) to permit that the medium term average spin rate (over decades to centuries) be consistent with the resonance. Even though this possibility cannot be excluded, there is up to now no evidence for such rapid fluctuations of the spin rate. Obviously, further measurements of eventual fluctuations in the spin rate of Venus over time would be of great interest to address this question. *Bills* [2005] showed that, due to the different dependencies of atmospheric and body tides on orbital eccentricity, long-period variations in the orbit of Venus will lead to variations in the net solar torque. He identified a number of higher order spin-orbit resonances in the vicinity of the present rotation rate of Venus, and argued that the tidal torque variations which are modulated by orbital eccentricity could move Venus through some of these resonances, with possible occasional capture. Up to now, however, no estimates of the strengths of those higher order resonances (which involve nonzero eccentricity) have been given.

[6] At this time, the question of whether there is resonance or not is therefore still open. However, the hypothesis that there is no resonance is difficult to sustain. As a matter of fact, in that case, the observed spin rate of Venus is uniquely governed by the balance between the solar atmospheric and body torques, irrespective of the resonance considerations. Nevertheless its period is within 0.1465 day from the period of the $p = -5$ resonance. Ignoring the higher order resonances, the resonances which are the closest to the observed spin rate, after the $p = -5$ resonance, are the ones with $p = -4.5$ and $p = -5.5$ which, according to equation (1), correspond respectively to $\omega_{4.5} = -2\pi/307.1107$ days and $\omega_{5.5} = -2\pi/201.2593$ days. Assuming for a while that the present rotation rate of Venus is determined independently of any consideration of spin-orbit resonance, the probability that the equilibrium period happens to be to be within 0.1465 days from a resonance can be written as

$$p_e = 2\frac{|\omega_S - \omega_5|}{|\omega_{4.5} - \omega_5|} = 0.58\%$$

Although not impossible, such a situation appears to be highly fortuitous. It is possible that, due to fluctuations of the orbital elements and/or solar flux, the equilibrium frequency may sweep through the resonances and Venus spin would sometimes be trapped, and then released. This however would not help since, in such a case, the time spent in a situation of resonance would further reduce the probability for the spin rate to be close, but not equal to the resonant rate.

[7] Considering this uncomfortable situation making it extremely difficult to avoid the role played by the spin-orbit resonance, this paper explores another hypothesis. According to this approach, I will assume that there is a $p = -5$ Earth-Venus resonance. However, assuming Venus is a system composed of a mantle, a liquid and a solid inner core, I propose that the resonance does not occur with the mantle, but with the solid core. The following parts of the paper attempt to determine the consequences and constraints resulting from this hypothesis.

2. Hypothesis of a Resonance Between the Earth and Venus's Core

[8] The hypothesis of a spin-orbit resonance between Venus's solid core and the Earth would permit us to understand why the rotation rate of the surface of Venus, as measured by either terrestrial or spaceborne radars, is very close, but not equal to the $p = -5$ resonance. It requires however a certain number of conditions to be fulfilled. First there should be a solid core with a spin rate different from the mantle's spin rate. Then the core-mantle spin difference should be within the allowable range. Also, the permanent gravitational quadrupole moment created by the solid core should be sufficient to permit the resonance to remain stable, but on the other hand the stresses within the core should not exceed the limit acceptable by the core material. Those questions will be addressed in this section.

2.1. Nature of Venus's Core

[9] The similarity in size and density between Venus and Earth suggests that both planets have similar internal structures involving a mantle surrounding a dense iron core. As concerns the Earth, seismological studies have revealed that the core is composed of a liquid outer core surrounding a solid inner core. It is classically believed that the terrestrial magnetic field is produced by dynamo action in the Earth's iron-rich fluid outer core. Fluid motions of the highly conducting liquid in the presence of the magnetic field induce currents which themselves generate the field. The most efficient process for driving the fluid motions in the liquid outer core is chemically driven convection. According to this process, buoyant light material is released as the liquid outer core freezes onto the solid inner core [*Braginsky*, 1964; *Stevenson et al.*, 1983]. This freezing process at the interface between inner and outer core is therefore believed to be responsible for the generation of convection, which in turn generates the terrestrial magnetic field.

[10] Contrary to the Earth, and despite its similarity in size, Venus does not possess a significant dipole magnetic field. Because of the aforementioned role of the inner-outer core interface in generating the Earth's magnetic field, this lack of a magnetic field at Venus has been interpreted by considering that the Venus core is either entirely liquid [*Stevenson et al.*, 1983] or completely solidified [*Arkani-Hamed and Toksöz*, 1984]. *Konopliv and Yoder*'s [1996] estimation of the k_2 potential Love number from Doppler tracking of Magellan and Pioneer Venus Orbiter, tended to favor the hypothesis of a liquid, rather than solid core for Venus. More recently, however, *Stevenson* [2003] released

Table 1. Torque Exerted by the Mantle on the Core, for Different Assumptions of Inner Core Radius and Outer Core Viscosity[a]

	R_1 = 1500 km	R_1 = 2000 km	R_1 = 2500 km
η = 470 Pa s	7.92×10^{12} Nm	2.34×10^{13} Nm	7.62×10^{13} Nm
η = 4700 Pa s	7.92×10^{13} Nm	2.34×10^{14} Nm	7.62×10^{14} Nm

[a] R_1, inner core radius; η, outer core viscosity.

those constraints, and concluded that there are two possible reasons why the liquid core of Venus does not convect. The first possibility is that there is no inner core. The other possibility is that the core is currently not cooling, and thus no freezing occurs at the inner core-outer core interface, chemically driven convection being therefore inhibited. According to this approach, such a regime began as Venus transitioned from a mobile surface to a stagnant lid regime, following a resurfacing event about 500 million years ago [*Schubert et al.*, 1997]. Such a difference with the Earth arises because the Earth has plate tectonics, which eliminates heat more efficiently than a stagnant lid form of mantle convection. Also, *Greff-Lefftz and Legros* [1999], and *Touma and Wisdom* [2001] developed models for core differentiated rotation on Venus (or the Earth), and explained how resonance passages in the past may have released energy to produce such resurfacing events of the planet.

[11] On the basis of the preceding discussion, in the remainder of this paper we shall assume that Venus's core consists of an inner and an outer core. While the outer core radius is fairly well known [*Stevenson et al.*, 1983], several assumptions will be tested as concerns the hypothetical inner core radius.

2.2. Differential Rotation Between Mantle and Inner Core and Associated Torque

[12] In the context of a spin-orbit resonance between Venus's inner core and the Earth, Venus is regarded as composed of two solid bodies (the mantle and the inner core) separated by the liquid outer core. Tidal dissipation occurs both within the mantle and within the core (including its inner and outer parts). We take the mantle spin rate as the one measured by *Davies et al.* [1992] from Magellan images of the surface, $\omega_M = -2\pi/243.0185$ days, while the inner core spin rate is assumed to be the $p = -5$ resonance spin rate $\omega_C = -2\pi/243.1650$ days. The differential rotation rate is therefore

$$\Delta\Omega = |\omega_M - \omega_c| = 1.74 \times 10^{-10} \text{ rad/s} = 0.31 \text{ degree/year}$$

[13] Such a differential rotation rate may be compared to the estimates of differential rotation rate obtained for the Earth by analyzing the travel times of seismic waves traversing the Earth's fluid and solid cores. Most estimates of the terrestrial inner core rotation rate are a few tenths of a degree per year faster than the rotation of the Earth (superrotation). Those estimates are however still rather uncertain, encompassing a large range of values, from zero rotation [*Souriau and Poupinet*, 2003], to intermediate values of 0.3 to 0.5 degrees/year [*Zhang et al.*, 2005] while estimates of more than 1 degree/year have also been reported [*Song and Richards*, 1996].

[14] One should note that for Venus, contrary to the Earth, the absolute value of ω_M is larger than the one of ω_C, which means that the retrograde rotation of Venus's mantle occurs faster than that of the core. This is not surprising, since for Venus the atmospheric torque tends to accelerate the planet, while the gravitational body torque tends to decelerate it. The accelerating atmospheric torque is transmitted to the mantle by friction at the planetary surface. Within the planetary body, part of this torque of atmospheric origin is canceled by the torque due to tidal friction within the mantle, while another part is canceled by the torque due to tidal friction within the core. Therefore, a net accelerating torque must be exerted by the mantle on the core (including its solid and liquid parts), which is the part of the torque of atmospheric origin that is not dissipated within the mantle, but merely transmitted by the mantle. It is not the purpose of this paper to identify how dissipation occurs within the complex system of the core including its solid and liquid parts. However, the torque required to maintain differential rotation may be evaluated as [*Bills*, 1999]

$$T_w = \alpha(R_1, R_2)\eta\,\Delta\Omega \tag{3}$$

where η is outer core viscosity, R_1 and R_2 are the radii of the inner and outer core, respectively, and $\alpha(R_1, R_2)$ is given by

$$\alpha(R_1, R_2) = 8\pi \frac{R_2^3 R_1^3}{R_2^3 - R_1^3} \tag{4}$$

For the Earth, the range of possible outer core viscosity values corresponding to seismic estimates of superrotation of the inner core was estimated by *Bills* [1999] as

$$470 \text{ Pa s} \leq \eta_o \leq 4700 \text{ Pa s} \tag{5}$$

[15] Lacking similar information concerning Venus, we shall use those bounds for the outer core viscosity of Venus. As concerns the outer core radius, *Stevenson et al.* [1983] reviewed several models of the core with radii ranging between 2890 km and 3110 km. Thus, taking R_2 = 3000 km will provide a reasonable order of magnitude. The inner core radius R_1 is unknown, and therefore different assumptions will be made here, with R_1 = 1500, 2000, and 2500 km, respectively. Table 1 gives the torque T_W exerted by the mantle on the core, depending on the assumption made for R_1 and η. This may be compared to the amplitude of the torque exerted by the atmosphere on the body, estimated as 1.8×10^{16} Nm by *Dobrovolskis and Ingersoll* [1980] under their heating at the ground hypothesis. One can see that, under that hypothesis, the torque from the mantle on the core represents only a very small portion of the atmospheric torque, from 0.04% to 4.2% depending upon the assumption taken for R_1 and η in Table 1, most of the torque transmitted by the atmosphere being thus tidally dissipated within the mantle in that case.

2.3. Observations of Gravity and Topography

[16] Both gravity and topography of Venus have been observed extensively by the Pioneer Venus Orbiter and

Magellan missions [e.g., *Sjogren et al.*, 1983; *Bills and Kobrick*, 1985; *Bills et al.*, 1987; *McNamee et al.*, 1993; *Nerem et al.*, 1993; *Konopliv et al.*, 1993], and correlations have been made between those measurements. *Bills et al.* [1987] reported that the correlation coefficient between gravity and topography is well above the 95% confidence level upper bounds on sample statistics from uncorrelated populations for every harmonic degree up to degree 15, except for the lowest degree harmonic (degree 2) for which the correlation coefficient, which is only of the order of 0.3, reveals no statistically significant correlation. This tends to indicate that the internal structure of the planet plays an important role on the $n = 2$ harmonic of the gravity field. Let us denote A, B, C the inertial moments of Venus, with $A < B < C$. *Konopliv et al.* [1993] compared the orientation of the axis of smallest inertia A to the orientations of the principal axes of the ellipsoid that best fits the Venus topography. Estimates of the longitude of axis of smallest inertia A range between −6.5° and −3.0° (hereafter we will take the average −4.7°), depending on the gravity model used. In contrast, among the 3 principal axes of the ellipsoid that best fits the Venus topography, the longest axis (with length 6052.214 km, compared to 6051.877 km and 6051.352 km for the two other axes) points toward a longitude of 281.1°. Orientations of the axes of smallest inertia based on either topography or gravity thus differ by 74.2 degrees. This large difference between the orientations of the gravitational and topographic axes is consistent with the lack of significant correlation between the $n = 2$ harmonics of topography and gravity. This is an indication that the direction of the axis of smallest inertia A is related to nonuniform density distributions within the planet. It is however not possible to determine whether those nonuniform density distributions are occurring within the mantle, or within the core, or result from nonaxisymmetric boundaries between outer core and mantle, or between inner and outer core. It is likely that the observed quadrupole moment of the planet results from a superposition of contributions from all those possible causes.

2.4. Condition for Stable Resonance

[17] In their theory of spin-orbit coupling in the solar system, *Goldreich and Peale* [1966] have studied in detail the conditions for Earth-Venus resonance between the spin and synodic periods of Venus. They conclude that the pth spin-orbit resonance will remain stable if any residual torque $\overline{T}$ about Venus's center of mass satisfies

$$|\overline{T}| < T_{\mathrm{Max}} = \frac{3}{2}K(p)(B-A)\left(\frac{GM_E}{a^3}\right) \tag{6}$$

where the overbar denotes an average over a (synodic) orbit period, G is the gravitational constant, M_E (=5.98 × 10^{24} kg) and a (=1.52 × 10^{11} m) are the terrestrial mass and semimajor axis, respectively, and $K(p)$ is a coefficient of restoring torque of the pth resonance ($K(p)$ = 2.531 for p = −5). *Goldreich and Peale* [1967] have further shown that viscous coupling between the rigid mantle and liquid core of Venus may supply the required dissipation allowing capture of Venus into the synodic resonance.

[18] From observations, the moment difference $(B - A)$ and C have been estimated, respectively, as [e.g., *Yoder*, 1997]

$$(B-A)/MR^2 = (2.22 \pm 0.01) \times 10^{-6},$$

$$0.331 \leq C/MR^2 \leq 0.341$$

where M (=4.87 × 10^{24} kg) and R(=6050 km) are Venus's mass and radius, respectively. This implies $(B - A)$ = 3.96 × 10^{32} kg m^2, $(B - A)/C \approx 6.5 \times 10^{-6}$, and T_{Max} = 1.72 × 10^{14} Nm.

[19] The moment difference $(B - A)_t$ that would be merely produced by a homogeneous ellipsoid fitting second harmonic topography with lengths of principal axes as given by *Konopliv et al.* [1993] (a_t = 6052.214 km, b_t = 6051.877 km, c_t = 6051.352 km) would be (see appendix A, equation (A3))

$$(B-A)_t = \frac{1}{5}\left(\frac{4}{3}\pi . a_t b_t c_t\right)\rho_t . \left(a_t^2 - b_t^2\right) \tag{7}$$

where the density ρ_t of the upper mantle is assumed here as uniform. Taking ρ_t = 3000 kg m^{-3}, this would give $(B - A)_t$ = 2.3 × 10^{33} kg m^2. This is more than 5 times larger than the observed value, $(B - A)$ = 3.96 × 10^{32} kg m^2, which indicates that matrixial summation of $(B - A)_t$ with other contributions reduces considerably the observed $(B - A)$. The other contributions come from inhomogeneities within the mantle (including isostatic compensation), within the inner core, as well as irregularities of the core-mantle and inner-outer core boundaries, although it is unfortunately not possible to distinguish between those contributions. Within the present model, we will merely consider that the observed $(B - A)$ consists of two components: one $(B - A)_M$ rotating with the mantle, and the other one $(B - A)_C$ rotating with the inner core. Since we are exploring in this paper the question of a resonance with the inner core, $(B - A)$ should be replaced by $(B - A)_C$ when applying inequality (6), as will be done below.

2.5. Condition on the Material Strength

[20] It is necessary to check whether the moment difference $(B - A)_C$ rotating with the inner core, which is required by the condition of stable resonance, can be supported by the material of the inner core. Evaluating the stress differences supported by the material of a planetary body as a function of topography, *Johnson and McGetchin* [1973] found that the maximum height h of topography on a planetary body could be estimated as

$$h \leq \frac{3kS}{4\pi G\rho^2 R} \tag{8}$$

where ρ and R are its density and radius, respectively. Here S is the ultimate strength of the material (the maximum stress it can withstand before failing), while k is a dimensionless coefficient between 1 and 3, depending on assumptions, including the variation of material strength with depth. The derivation of equation (8) above by *Johnson and McGetchin* [1973] is reproduced in Appendix B of this paper (compare equation (B4)).

[21] As can be seen in Appendix B, in the case of a solid core embedded within a less dense liquid outer core, equation (8) should be replaced by equation (B6), which involves the difference $\Delta\rho$ of densities of the solid and liquid phases at the inner core-outer core interface, and which may be rewritten, for the inner core with radius R_1 and density ρ_1:

$$h \leq \frac{3kS}{4\pi G\rho_1.\Delta\rho.R_1} \tag{9}$$

In order to transform this constraint into a constraint on $(B - A)/C$, we may model the inner core as an homogeneous ellipsoid with principal semiaxes a_1, b_1, c_1, and corresponding inertial moments A_1, B_1, C_1. As recalled in equation (A1) of Appendix A, these are related to the principal semiaxes, and mass M of the ellipsoid through

$$A_1 = \frac{1}{5}M.\left[b_1^2 + c_1^2\right];\ B_1 = \frac{1}{5}M.\left[a_1^2 + c_1^2\right];\ C_1 = \frac{1}{5}M.\left[a_1^2 + b_1^2\right] \tag{10}$$

Therefore, one gets

$$\frac{B_1 - A_1}{C_1} = \frac{a_1^2 - b_1^2}{a_1^2 + b_1^2} \tag{11}$$

where the inertial moment C_1 is obtained through

$$C_1 \approx \frac{2}{5}\left(\frac{4}{3}\pi R_1^3\rho_1\right)R_1^2 \tag{12}$$

[22] When the inner core is embedded within an otherwise spherically symmetric liquid core, the same expression holds, except that the mass anomaly produced by surface topography is not proportional to the inner core density ρ_1, but rather to the solid-liquid density difference $\Delta\rho$ at the solid-liquid interface. Although the precise values of core densities are not available for Venus, reasonable orders of magnitude estimates for core densities may be obtained from the preliminary reference earth model (PREM) [*Dziewonski and Anderson*, 1981], from which we assume here $\rho_1 \approx 12.8 \times 10^3$ kg m^{-3}, and $\Delta\rho \approx 0.60 \times 10^3$ kg m^{-3}. From equation (11) combined with equation (A7) of Appendix A, we get that the mass anomaly at the solid-liquid core interface produces the following moment difference $(B - A)_C$:

$$\frac{(B - A)_C}{C_1} = \frac{a_1^2 - b_1^2}{a_1^2 + b_1^2} \cdot \frac{\Delta\rho}{\rho_1} \tag{13}$$

This quantity accounts for density anomalies rotating with the inner core. It does not take into account the contribution from the mantle, or the contribution from the mantle-outer core interface, whose density anomalies are expected to rotate instead with the mantle.

[23] Identifying maximum topography of the inner core as $h = (a_1 - b_1)/2$, one obtains the following condition from (9) and (13), to first order:

$$\frac{(B - A)_C}{C_1} \leq \frac{3kS}{2\pi G\rho_1^2 R_1^2} \tag{14}$$

or equivalently (from equation (12))

$$(B - A)_C \leq \frac{4}{5}\frac{kSR_1^3}{G\rho_1} \tag{15}$$

Here we have simplified the problem by taking a homogeneous ellipsoidal inner core. The mass anomaly may also be produced by a more complex shape, as well as heterogeneities within the inner core. In any case the stress differences that must be supported by the material would be related to $(B - A)_C$ and therefore one can expect inequality (15) also to address adequately those more complex situations.

2.6. Gravitational Coupling Between Core and Mantle

[24] We have ignored so far the gravitational torque that may be experienced by Venus's inner core from the mantle due to misalignments of their principal axes from their positions of static equilibrium. The inner core would experience no gravitational torque from the mantle in the case where Venus's mantle were a perfect spherical shell, for example, or more generally if its mantle were a « homeoid » bounded by two similar coaxial ellipsoids [e.g., *MacMillan*, 1958]. In either case the gravitational force and torque of the mantle on the inner core then vanish (even though the gravitational anomaly does not vanish outside the planet). If however such a specific situation is not fulfilled, a major problem with considering the inner core and mantle as rigid bodies is that gravitational coupling between them is then likely to be sufficiently strong to prevent differential rotation from happening. Long after the pioneering work by *Poincaré* [1910] on the dynamics of the terrestrial body involving a liquid core, *Buffett* [1996a, 1996b] computed the gravitational torque exerted by misalignment of inner core and mantle from their positions of static equilibrium, assuming that they behave as rigid bodies. In that case he obtained gravitational torques with amplitudes as high as 10^{21} Nm for the Earth, which would be sufficient to keep the inner core aligned with the mantle. From *Buffett*'s [1996a, 1996b] approach, *Xu et al.* [2000] gave the following expression for the gravitational torque:

$$T_{CM} = \frac{4\pi G}{5}\left[\int_{R_1}^{R} \rho(r')\frac{\partial\beta}{\partial r'}dr' + \rho_f\beta_1\right]\left[(B_1 - A_1) - \left(B_1' - A_1'\right)\right] \cdot \sin 2\delta\varphi \tag{16}$$

where $\delta\varphi$ is the azimuthal deviation of the mantle with respect to the inner core. A'_1, B'_1 are the equatorial moments of a body with the shape of the inner core and density of the fluid outer core. The equatorial flattening β is defined as the relative difference between the equatorial radii of the surfaces of constant densities (assumed ellipsoidal). Here β_1 is the equatorial flattening of the inner core, while ρ_f is the outer core density at the inner core boundary.

[25] *Van Hoolst et al.* [2008] gave the expression of that gravitational torque in the simple case of a system of three homogeneous layers, involving a solid outer shell surrounding a liquid intermediate body and a solid inner body, assuming each of these layers to have a constant density. In

that case they obtained the following expression for the gravitational torque:

$$T_{CM} = \frac{4\pi G}{5}[\rho_M\beta_M + (\rho_o - \rho_M)\beta_o]\left(\frac{\Delta\rho}{\rho_1}\right)(B_1 - A_1)\sin 2\delta\varphi = T_{CMo}\sin 2\delta\varphi \quad (17)$$

Here β_o and β_M are the equatorial flattenings of the outer core and mantle outer boundaries (assumed ellipsoidal). Also, ρ_1 and ρ_o are the densities of the inner and outer cores, respectively, while ρ_M is the density of the mantle.

[26] Note that equation (17) was obtained assuming that the liquid outer core has a uniform density (in fact the liquid intermediate medium of *Van Hoolst et al.*'s [2008] study was an aqueous ocean with uniform density). In contrast, in our case the density of the liquid outer core at its lower boundary is presumably different from ρ_o due to compressibility of liquid iron, since *Dziewonski and Anderson*'s [1981] terrestrial model gives a density difference as high as 2.5×10^3 kg m^{-3} between the lowermost and uppermost outer core's densities. We will however use equation (17) here for a while because of its simple formulation compared to equation (16), in order to discuss orders of magnitude estimates of the various contributions to the gravity anomaly, taking $\Delta\rho$ as the density difference at the inner core-outer core boundary ($\Delta\rho = 0.6 \times 10^3$ kg m^{-3}).

[27] If T_{CMo} is not too strong, the effect of the torque T_{CM} will be an oscillatory behavior of the inner core rotation rate around the resonant rate $\omega_r = 2\pi/243.1650$ days, as will be shown below. Correlatively, the core-mantle differential rate $\delta\Omega$ will oscillate around the average differential rate $\Delta\Omega$ = 0.31degree/year mentioned in section 2.2.

[28] Assuming for simplicity that the inertial momentum of the inner core is much smaller than the mantle inertial momentum, and ignoring here viscous coupling, the variations of $\delta\Omega$ will be governed by the following coupled differential equations:

$$\begin{aligned} C_1 . d(\delta\Omega)/dt &= T_{CMo}\sin(2\delta\varphi) \\ d(\delta\varphi)/dt &= \delta\Omega(t) \end{aligned} \quad (18)$$

where C_1 is the inertial momentum of the inner core. Eliminating time we get a differential equation between $\delta\Omega$ and $\delta\varphi$ alone, whose integration is straightforward and gives

$$(\delta\Omega)^2 = -\frac{T_{CMo}}{C_1}\cos(2\delta\varphi) + \mu \quad (19)$$

where μ is a constant of integration. The first term of the right hand side accounts for the oscillatory behavior of $(\delta\Omega)^2$. Since the left hand side of equation (19) is positive definite, $\delta\varphi$ will be free to take any value between 0 and 2π provided that the right hand side of equation (19) remains positive, i.e., if $\mu > 0$ and $|T_{CMo}|/C_1 < \mu$. Otherwise, $\delta\varphi$ will be constrained to remain within a limited interval, and therefore the inner core will be trapped by the mantle's rotation. If there is resonance, the average of $\delta\Omega$ is $\Delta\Omega$, and thus the value of μ is close to $(\Delta\Omega)^2$. We conclude that for resonance to be possible the amplitude $|T_{CMo}|$ should satisfy the following condition:

$$\frac{|T_{CMo}|}{C_1} < (\Delta\Omega)^2 \quad (20)$$

[29] For example for an inner core radius of 2000 km, this condition gives $|T_{CMo}| < 2.1 \times 10^{16}$ Nm. This limit should be compared to the value of T_{CMo} given in equation (17). If one takes $(\Delta\rho/\rho_1)(B_1 - A_1) = 3.96 \times 10^{32}$ kg m^2 from the observed semidiurnal component of Venus gravity as discussed in section 2.4, equation (17) gives

$$T_{CMo} = [\rho_M\beta_M + (\rho_o - \rho_M)\beta_o] \times 6.6 \times 10^{22}\,\text{Nm} \quad (21)$$

where densities are expressed in kg m^{-3}. From the terrestrial estimates [*Dziewonski and Anderson*, 1981], we will take $\rho_M = 3.0 \times 10^3$ kg m^{-3} for the mantle and $\rho_o = 9.8 \times 10^3$ kg m^{-3} for the upper layers of the outer core. The quantity β_M can be estimated from the lengths of principal axes of topography given in section 2.4, which give $\beta_M = (a_t - b_t)/a_t \approx 5.6 \times 10^{-5}$. If the flattening of the core-mantle boundary were ignored (β_o = 0), equation (21) would yield $T_{CMo} = 1.1 \times 10^{22}$ Nm. This is 5.2×10^5 times larger than the upper limit of 2.1×10^{16} Nm discussed above from condition (20). Therefore in order for differential rotation to be possible, the second term within the bracket of equation (21) should be negative and virtually equal to the first one to within less than 2.10^{-6}. Opposite signs of β_M and β_o mean that, in the equatorial plane, the minor axis of the mantle inner boundary should be directed along the major axis of the planetary surface (i.e., deeper inner boundary associated with higher topography), and this behavior should compensate almost entirely the gravity anomaly within the core.

[30] Thus, it appears that differential rotation between the inner core and mantle will be impossible, unless some very efficient compensation mechanism shields the inner core from the semidiurnal gravity anomalies originating within the mantle. It is proposed here that isostatic compensation, leading to hydrostatic equilibrium below the compensation depth, provides such a mechanism acting to cancel those gravity anomalies in the deep interior. In the simplistic three-layer model described above, the density was assumed constant within the mantle and compensation was expected to be achieved by the shape of the CMB. In reality isostatic compensation is believed to occur principally in the crust (i.e., within the first few tens of km below the planetary surface) and also including parts of the upper mantle, and loads with half width >500 km on Earth are believed to be in approximate isostatic equilibrium [e.g., *Keary and Vine*, 1996]. It should be noted that the assumption of isostatic compensation with an effectiveness of 99.9998%, as discussed above, is a very stringent hypothesis, and the possibility of such high level of compensation is rather speculative. For example, a crust with thickness variations of 30 km having this level of compensation could not sustain an uncompensated load more than 6 cm in amplitude without violating the condition. Such a hypothesis is however needed in the framework of this paper. Of course, this does not exclude the possibility of compensation also

occurring at the core-mantle boundary (CMB), and *Schubert et al.* [2001] propose that broad-scale undulations of the D″ layer (at the base of the mantle) and the CMB probably represent dynamic topography, which would be expected to relax on a short time scale considering the rheology at the high temperatures in D″. Finally, hydrostatic equilibrium is expected to apply within the fluid outer core. Hydrostatic equilibrium below the compensation depth does not however mean that anomalies of the gravitational potential vanish in the deep interior. In this paper, we need to assume that, in the coupled system involving lithosphere, mantle and fluid compressible outer core, the flattening function $\beta(r)$ will adjust toward a state of minimum energy, which would reduce considerably the gravity anomaly at depth, and correlatively the factor in brackets in equation (16). How high is the residual? This paper will unfortunately not resolve this difficult question, and further realistic modeling of the gravitational potential semidiurnal anomaly below a fully compensated load, involving a multilayer approach including lithosphere, mantle, and radial density gradient in the compressible fluid outer core, would be required to answer that question. Such modeling should compute in a self-consistent manner the radial profile of equatorial flattening $\beta(r)$ to be used in equation (16).

2.7. Consequences on the Balance of the Atmospheric and Body Tidal Torques

[31] Henceforth in this paper we will assume that isostatic compensation exists with an effectiveness of 99.9998% at the planetary scale, as discussed in section 2.6. The balance between the atmospheric and body tidal torques at Venus (respectively T_A and T_B) has been studied by several authors, and long-term simulations were performed for the evolution of the spin of Venus under the effect of solid tides, atmospheric tides, core-mantle friction and planetary perturbations [*Dobrovolskis and Ingersoll*, 1980; *Dobrovolskis*, 1980; *Correia and Laskar*, 2001, 2003; *Correia et al.*, 2003]. Within the constant Q assumption, and heating at the ground model, *Dobrovolskis and Ingersoll* [1980] estimate these torques as

$$\begin{aligned} T_B &\approx -\frac{3}{2} G M_S^2 \left(\frac{R_V^5}{a_V^6}\right) \frac{k_L}{Q}, \\ T_A &= \frac{3\pi}{8} \frac{M_S}{M_V} \left(\frac{R_V^6}{a_V^3}\right) \frac{\kappa F_o}{H_o \sigma} = T_0 \frac{\sigma_e}{\sigma} \end{aligned} \tag{22}$$

where M_S and M_V are solar and Venusian masses, R_V and a_V are Venus's radius and solar distance, respectively, $\kappa = R/c_p \approx 0.2$, F_o is the amount of solar flux absorbed at the ground, H_o is the atmospheric scale height at the ground, k_L and Q are Venus's Love number for potential and tidal quality factor of the body torque, σ is the semidiurnal frequency of Venus, and the equilibrium frequency σ_e is the value of semidiurnal frequency at which the atmospheric and body torques cancel out each other perfectly. The value of the body torque T_B, however, is a rather unknown quantity. From equations (22), and assuming F_o = 100 W m^{-2}, *Dobrovolskis and Ingersoll* [1980] obtain $T_B \approx -2.0.10^{18}$. (k_L/Q) Joules, $T_A \approx T_o \approx 1.8.10^{16}$ Joules. Assuming $k_L \approx 0.25$, this corresponds to $Q = 28$. It is consistent with the estimated values of Q for terrestrial planets, which usually range from 10 to 100. However, Q might not necessarily be independent from tidal frequency, as discussed by *Dobrovolskis* [1980], who proposed alternative models to the constant Q model, such as a viscous model where Q is inversely proportional to tidal frequency. In such case, since Venus is rotating very slowly, Q for Venus might be much higher than for the other terrestrial planets. Also, as reported by *Dobrovolskis and Ingersoll* [1980], depending upon the altitude where most absorption of solar radiation occurs, the imaginary part of the semidiurnal surface pressure variation $\mathrm{Im}(\delta p_o^{\sigma,2})$ (to which the atmospheric torque is proportional) may be much smaller than the one obtained with the heating at the ground model. Keeping the approach of heating in the lower atmosphere, but with other assumptions for the distribution of heating with height, they obtained $\mathrm{Im}(\delta p_o^{\sigma,2})$ as low as about 4 times smaller than the heating at the ground model, while assuming absorption only in the upper atmosphere reduced $\mathrm{Im}(\delta p_o^{\sigma,2})$ by a factor of 100 or more. *McCue et al.* [1992] estimated the viscous torque exerted by the Venusian atmosphere on the body of the planet as 0.68×10^{14} Joules, and argued that this value is comparable to the atmospheric torque, which they estimate as 0.44×10^{14} Joules.

[32] Due to these uncertainties of both the atmospheric and body tides, in the following we will use *Dobrovolskis and Ingersoll*'s [1980] heating at the ground estimate (with 100 W m^{-2} of solar flux absorbed by the ground) $T_o = T_{og} = 1.8.10^{16}$ Joules as a reference (corresponding to $Q = 28$), but we will also discuss the consequences if T_o were reduced by a factor of 100 compared to this estimate (corresponding to $Q = 2800$).

[33] In the absence of a resonance, the stationary situation is obtained for $T_B = -T_A$, and therefore $\sigma = \sigma_e$. On the contrary, if the $p = -5$ resonance occurs, then one gets

$$\sigma = \sigma_5 = 2.(n_V - \omega_R) = 2.(2\pi/116.78 \text{ days})$$

and the residual torque $\delta T = T_A + T_B$ can thus be written

$$\delta T = -T_0 \left(1 - \frac{\sigma_e}{\sigma_5}\right) \tag{23}$$

Let us introduce parameter λ as

$$\lambda = \frac{\sigma_e - \sigma_5}{\sigma_{4.5} - \sigma_5} \tag{24}$$

where $\sigma_{4.5}$ is the resonance semidiurnal frequency at $p = -4.5$. Parameter λ characterizes a relative distance of the equilibrium frequency σ_e from the closest resonant frequency σ_5. Equation (23) can be rewritten

$$\delta T = -\frac{\sigma_5 - \sigma_{4.5}}{\sigma_5} \lambda T_0 = -\frac{1}{10} \lambda T_0 \tag{25}$$

Condition for stable resonance of the inner core can now be rewritten by requiring $|\delta T| \le T_{\max}$, where $T_{\max}$ is given by equation (6) (with $(B - A)$ replaced by $(B - A)_C$). This gives

$$\frac{1}{10} |\lambda| T_0 \le \frac{3}{2} K(-5)(B - A)_C \left(\frac{GM_E}{a^3}\right) \tag{26}$$

Inequality (26) gives a lower limit for $(B - A)_C$ produced by the irregular shape and/or inhomogeneous density of the inner core. It is then necessary to check whether the inner core material may have a strength sufficient to sustain the stress differences resulting from such spherically asymmetrical mass distributions. Thus condition (26) must be compatible with the constraint on the strength of the inner core material (condition (15)). Eliminating $(B - A)_C$, we get the condition that λ must fulfill in order that both constraints be compatible:

$$|\lambda| \leq \lambda_M = 12K(-5)\frac{kSM_E}{a^3 T_0}\frac{R_1^3}{\rho_1} \quad (27)$$

[34] It is necessary to evaluate the strength S of the material of the solid inner core. Iron is considered to be the dominant component of the Earth and Venus cores, and iron in the Earth's inner core is generally believed to be in the hexagonal close packed (hcp) phase [*Song*, 1997, and references therein], although the possibility that it is in the body centered cubic (bcc) phase has also been proposed [*Dubrovinsky et al.*, 2007]. *Singh et al.* [2006] have investigated the strength of iron under pressure up to 55 GPa. Even though those conditions are still far from the ones prevailing at Earth's inner core (pressure of the order of 330 to 360 GPa), they could observe the transition from the bcc to the hcp phases that, for the conditions of their experiment, occurred in the pressure range 12–18 GPa. Below the transition *Singh et al.* [2006] obtained compressive strength of bcc iron (or α-Fe) of the order of S = 1.1 GPa, while above the transition they obtained a compressive strength of hcp iron (or ε-Fe) showing a monotonic increase with increasing pressure, fitting the relation $S = 2.9 + 0.028P$, where compressive strength S and pressure P are expressed in GPa, respectively. Although the pressure in Venus's core is expected to be somewhat lower than the one at Earth's inner core, the iron of a hypothetical inner core at Venus is still expected to be in either the hcp or bcc phases. Due to those uncertainties, we take the lower estimate of S = 1 GPa, assuming that it gives a reasonable order of magnitude estimate for the strength of the core material. Also, ρ_1 is taken as 12.8×10^3 kg m^{-3}, and coefficient k is set to its most constraining value k = 1. These assumptions allow condition (27) to be computed. Introducing the outer core radius R_2 = 3000 km, and the atmospheric torque obtained within the heating at the ground hypothesis $T_{og} = 1.8 \times 10^{16}$ Joules, it can be rewritten

$$|\lambda| \leq \lambda_M = 6.1\left(\frac{T_{og}}{T_o}\right)\cdot\left(\frac{R_1}{R_2}\right)^3 \quad (28)$$

[35] The equilibrium frequency σ_e depends only on the relative efficiencies of the atmospheric and body tides, regardless of the positions of the resonance frequencies. Given a value of σ_e, assumed chosen randomly, the value of parameter λ for the closest resonance (which for the case on hand occurred to be the $p = -5$ resonance) is necessarily comprised between −0.5 and +0.5, while the probability density for λ is uniformly distributed within the [−0.5, +0.5] interval. Therefore, as soon as λ_M is larger than 0.5, condition (28) will be fulfilled, without any further condition on the position of equilibrium frequency σ_e relative to the series of resonances.

[36] Within the heating at the ground hypothesis ($T_o = T_{og}$), this condition $\lambda_M \geq 0.5$ can be rewritten $R_1/R_2 \geq 0.43$, or equivalently $R_1 \geq 1300$ km. If $R_1 < 1300$ km, stability of resonance is still possible, but it requires that σ_e be fortuitously close to a resonance. The probability that the position of σ_e is compatible with stable resonance is $p_S = 2\lambda_M$. For example, from equation (28), probability p_S of more than 10% requires $R_1 \geq 600$ km.

[37] If now we assume that the atmospheric torque is smaller by a factor of 100 than the one obtained with the heating at the ground assumption, then $T_o = T_{og}/100$ and condition (28) yields $R_1/R_2 \geq 0.094$, or equivalently $R_1 \geq 280$ km. In that case, from equation (26) and assuming the least favorable case $|\lambda| = 0.5$, we obtain that the minimum value of $(B - A)_C$ for stable resonance should be $(B - A)_C \geq 2.1 \times 10^{31}$ kg m^2, which is no more than 5.3% of the observed moment difference $(B - A)$ of Venus (see section 2.4). Thus in that case even a relatively weak moment difference of the inner core, compared to the moment difference of the whole planet, would be sufficient to ensure stable resonance of the core.

3. Conclusion and Discussion

[38] The observation that the spin period of Venus is very close to, although not equal to the $p = -5$ spin-orbit resonance with the Earth, makes it very improbable that such a situation is fortuitous. Indeed, if the spin rate resulted merely from the balance between the atmospheric and body tides, without interference from the spin-orbit resonance with the Earth, the probability for the spin rate to be so close to one of the spin-orbit resonances would be less than 0.6%. Although not impossible, this highly improbable situation leads one to explore alternative hypotheses in which the Earth spin-orbit resonances play a role in the observed spin rate of Venus. This paper proposes such a hypothesis. According to this model, Venus's core is composed of a liquid outer core surrounding a solid inner core. The inner core and the mantle are thus a system of two coupled solid bodies spinning with slightly different angular frequencies, and spin-orbit resonance occurs not with the mantle, but with the inner core. The observed spin rate of the surface of Venus reflects the spin rate of the mantle, rather than the resonant spin rate of the inner core, and this would explain why the observed spin rate is different from, albeit very close to the resonant one. The constraints following from this hypothesis are analyzed in this paper.

[39] Within the present model, the lack of a magnetic field at Venus today implies that freezing of the outer core onto the inner core, which is expected to produce chemically driven convection responsible for the generation of the magnetic field at Earth, is not occurring at Venus. This would indicate that, contrary to Earth, the Venus core is currently not cooling, a situation which could have begun as Venus transitioned from a mobile surface to a stagnant lid regime, following a resurfacing event about 500 million years ago [*Schubert et al.*, 1997; *Stevenson*, 2003].

[40] The torque transmitted by the mantle to the core by the spin difference of 0.31 degree/year can be estimated depending on the assumed outer core viscosity and inner

core radius. Using the range of outer core viscosities estimated for the Earth ($470 \leq \eta_o \leq 4700$ Pa s), and for inner core radius in the range 1500–2500 km, this torque would amount to only 0.04% to 4.2% of the accelerating torque of atmospheric tides, estimated as $\approx 1.8 \times 10^{16}$ Nm by *Dobrovolskis and Ingersoll* [1980] within their heating at the ground model. In that case, most of the atmospheric accelerating torque transmitted to the body would thus be tidally balanced within the mantle.

[41] An inner core with such a strong gravitational quadrupole moment would undergo such high gravitational coupling that differential rotation with the mantle would probably not be possible, unless some very efficient compensation mechanism shields the inner core from the gravity anomalies originating within the mantle. It is proposed that isostatic compensation provides such a mechanism, acting to cancel those gravity anomalies in the deep interior. However, further realistic modeling of the semidiurnal anomaly of gravitational potential below a fully compensated load would be required to assess that point. It might also be interesting to explore alternative hypotheses where the inertia tensor of some component of Venus's gravitational field would rotate at the resonant rate, without invoking core-mantle differential rotation. This might be accomplished through internal convection, for example. An analogy would be the westward drift of Earth's magnetic field. Such hypotheses are however outside the scope of this paper.

[42] The ability of a planetary body to be trapped into stable spin-orbit resonance depends on the difference ($B - A$) between its two smallest principal moments of inertia [*Goldreich and Peale*, 1966]. In the context of our model, the ($B - A$) which is observed by orbiting spacecraft results from the combination of a contribution $(B - A)_M$ of the external body rotating with the mantle, and a contribution $(B - A)_C$ of the inner body rotating with the inner core. Therefore the condition for stable resonance of the inner core requires $(B - A)_C$ to be sufficiently important. In other words, the inner core should depart significantly from spherical symmetry, due to either an elongated shape and/or density heterogeneities. On the other hand, the material of the inner core should be able to sustain the stress differences produced by such asymmetric shape and/or density, and this imposes an upper limit of the allowable $(B - A)_C$. Compatibility of those two conditions can be achieved provided that the inner core average radius R_1 is sufficiently large. The minimum value for R_1 is determined in this paper, assuming the strength of the inner core material to be of the same order as either 'bcc' or 'hcp' type iron. The minimum R_1 also depends on the amplitude, at the resonant spin rate, of the residual torque resulting from the cancellation between atmospheric and body tides. The closer the equilibrium spin rate to a resonant spin rate, the smaller the residual torque and the easier the stable resonance. Apart from the $p = -5$ resonance there are a series of resonant spin rates with p integer multiples of 1/2. In the less favorable situation (i.e., equilibrium spin rate equally distant from $p = -5$ and $p = -4.5$ resonant spin rates), and in the context of a heating at the ground model for the atmospheric thermal tide, we found that the different constraints mentioned above may be compatible provided that $R_1 \geq 1300$ km. If, on another hand, one assumed that absorption of solar radiation were occurring in the upper atmosphere rather than near the ground, then that condition would be considerably less stringent and much smaller values of R_1 could be allowed.

[43] Spherical harmonic models of Venus's gravity field have been produced from the tracking data sets from the Pioneer Venus Orbiter (PVO) and Magellan missions. Data from PVO were obtained from 1978 to 1992, while data from Magellan were obtained from 1990 until 1994. Available gravity field models are therefore based on observations performed over a time interval of 16 years. If both mantle and inner core contribute significantly to the $n = 2$ harmonics, it would be therefore tempting to try to identify some variation of the $n = 2$ harmonics in relation with mantle-core differential rotation over that 16 year time interval. Unfortunately, a differential rotation rate of 0.31 degrees/year between mantle and inner core would have only produced a differential rotation of less than 5 degrees over this time range. The angular drift of the total planetary gravity field resulting from the superposition of mantle-related and core-related mass anomalies would have been even smaller, especially if the moment difference $(B - A)_C$ of the core only represents a minor contribution to the total observed moment difference ($B - A$), as may be the case if atmospheric heating occurs primarily in the upper atmosphere. In addition, because accurate determination of harmonic coefficients of the gravity field requires data sets involving as many as possible situations of latitude, longitude and altitude of the spacecraft, the lowest order harmonics of the most recent gravity models were produced by combining all available measurements covering the whole time of operation of both PVO and Magellan [e.g., *Barriot et al.*, 1998; *Konopliv et al.*, 1999]. It is therefore difficult at this time to identify long-term time variations of the $n = 2$ spherical harmonic coefficients. Independent measurements of the $n = 2$ harmonic coefficients from the Venus Express Radio Science experiment [*Häusler et al.*, 2006] may be useful to give an estimate of the $n = 2$ harmonics for the period 2006–2009, while waiting for further measurements in several decades which might allow us to address that question unambiguously.

Appendix A: Inertial Moment Difference of an Ellipsoidal Homogeneous Body Embedded Within an Otherwise Spherically Symmetric Liquid Outer Core

[44] We first consider an homogeneous ellipsoid with density ρ and principal semiaxes a, b, c (with $c \leq b \leq a$), with corresponding inertial moments A, B, C. Classical integration gives the following relations between the inertial moments, principal semiaxes, and mass M of the ellipsoid:

$$A = \frac{1}{5}M\left(b^2 + c^2\right); B = \frac{1}{5}M\left(a^2 + c^2\right); C = \frac{1}{5}M\left(a^2 + b^2\right) \quad \text{(A1)}$$

with

$$M = \frac{4}{3}\pi abc\rho \quad \text{(A2)}$$

The difference between the two smallest inertial moments B and A is therefore

$$B - A = \frac{1}{5}\left(\frac{4}{3}\pi abc\right)\rho\left(a^2 - b^2\right) \quad \text{(A3)}$$

Assuming an homogeneous liquid sphere with density ρ_o, its inertial moments A_o, B_o, C_o along any three orthogonal directions will be equal: $A_o = B_o = C_o$. Suppose that, within this liquid body, one removes a centered ellipsoid with principal semiaxes a_1, b_1, c_1 (with $c_1 \leq b_1 \leq a_1$), and replaces it by an homogeneous solid ellipsoid of the same shape, but with density ρ_1. The inertial moments of the composite body (including liquid and solid parts) will then be

$$\begin{aligned} A_C &= A_o + \frac{1}{5}\left(\frac{4}{3}\pi a_1 b_1 c_1\right)(\rho_1 - \rho_o)\left(b_1^2 + c_1^2\right) \\ B_C &= A_o + \frac{1}{5}\left(\frac{4}{3}\pi a_1 b_1 c_1\right)(\rho_1 - \rho_o)\left(a_1^2 + c_1^2\right) \\ C_C &= A_o + \frac{1}{5}\left(\frac{4}{3}\pi a_1 b_1 c_1\right)(\rho_1 - \rho_o)\left(a_1^2 + b_1^2\right) \end{aligned} \quad \text{(A4)}$$

and thus, for the composite body

$$B_C - A_C = \frac{1}{5}\left(\frac{4}{3}\pi a_1 b_1 c_1\right)\Delta\rho\left(a_1^2 - b_1^2\right) \quad \text{(A5)}$$

where $\Delta\rho = (\rho_1 - \rho_o)$.

[45] For the inner solid body, equation (A3) implies

$$B_1 - A_1 = \frac{1}{5}\left(\frac{4}{3}\pi a_1 b_1 c_1\right)\rho_1\left(a_1^2 - b_1^2\right) \quad \text{(A6)}$$

Combining equations (A5) and (A6), we get

$$B_C - A_C = \frac{\Delta\rho}{\rho_1}(B_1 - A_1) \quad \text{(A7)}$$

If the inner solid ellipsoid is very close to a sphere with radius R_1 (with $(a_1 - c_1)/R_1 \ll 1$), and if instead of an homogeneous distribution $\rho = \rho_o$ the outer liquid distribution is replaced by a spherically symmetric distribution ($\rho = \rho_o$ for $r \leq a_1$, but $\rho = \rho(r)$ for $r > a_1$), this would be equivalent to adding spherically symmetric shells whose contribution to the difference $(B_C - A_C)$ would be zero, and thus equation (A7) would still hold.

Appendix B: Maximum Height of Inner Core Topography

[46] In order to estimate the scale of topography permitted by the strength of the material of a planetary body, *Johnson and McGetchin* [1973] considered the simple case of a spherical, nonrotating, incompressible object of radius R, mass m, and uniform density ρ. Surface gravity g is given by

$$g = \frac{Gm}{R^2} = \frac{4}{3}\pi\rho GR \quad \text{(B1)}$$

The surface load or principal stress directed radially toward the planet's center of gravity by topography of height h is then simply

$$P = \rho gh = \frac{4}{3}\pi\rho^2 GRh \quad \text{(B2)}$$

[47] *Johnson and McGetchin* [1973] argue that the maximum stress difference due to the load, $\Delta\sigma_m$, may be estimated as $\Delta\sigma_m = P/k$, where coefficient k ranges between 1 and 3. Assuming that failure occurs when the maximum stress difference $\Delta\sigma_m$ exceeds the strength of the material, S, they can estimate the maximum scale of topography allowed from the relation

$$P/k \leq S \quad \text{(B3)}$$

From equation (B2), this condition (B3) can be rewritten

$$h \leq \frac{3kS}{4\pi G\rho^2 R} \quad \text{(B4)}$$

[48] In the case of a solid core imbedded within an otherwise spherically symmetric liquid outer core, the stress difference due to the load on various locations of the inner core is related not to the density of the inner core, but rather to the difference $\Delta\rho$ of densities of the solid and liquid phases at the inner core-outer core interface. As a consequence, in that case equation (B2) should be replaced by

$$P = \Delta\rho.gh = \frac{4}{3}\pi\rho.\Delta\rho.GRh \quad \text{(B5)}$$

Therefore in that case, combining condition (B3) with equation (B5), one gets

$$h \leq \frac{3kS}{4\pi G\rho.\Delta\rho.R} \quad \text{(B6)}$$

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